Supporting Information for "Probing the Sources of Uncertainty in Transient Warming on Different Time-Scales"

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1. Text S1 and S2

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2. Tables S1 to S3

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Introduction The supplementary material contains two text sections, three tables and one figure. The first text section provides details on how the EBM was fit to the CMIP5 data. The second text section describes where the historical forcing data was obtained from and how the EBM was fit to historical global-mean surface temperatures. The first table defines the terms in the analytic solution to the EBM (equations 5 and 6 of the main text); the second table lists the CMIP5 models used in this study and their corresponding values of F, λ, c, c_0, γ, ϵ, TCR and ECS; and the third table shows r^2 values for correlations between the six parameters of the EBM across the CMIP5 models. The figure compares time-series of global-mean surface temperature from CMIP5 model runs with CO_2 concentrations increasing by 1% per year and fits of the EBM to these simulations.

Text S1. Fitting and integrating the energy balance model The EBM was fitted to each model using the three step procedure of *Geoffroy et al.* [2013a]. This method consists of two steps to fit the EBM, assuming the ocean heat uptake efficacy $\epsilon = 1$, and then a third step in which ϵ is adjusted.

In the first step, F and λ are estimated using the *Gregory et al.* [2004] method by linearly regressing the net TOA radiative imbalance (R) in the quadrupled CO_2 experiments versus the global-mean surface temperature (T). The forcing F is then equal to the y-intercept of the regression and λ is equal to the slope of the regression.

For $t >> \tau_F$, the solution to the EBM can be written as

$$T_1 \approx ECS(1 - a_s e^{-t/\tau_s}), \tag{1}$$

and so

$$log(1 - \frac{T_1}{ECS}) \approx log(a_s) - \frac{1}{\tau_s} t. \tag{2}$$

Provided that $\tau_F << 30$ years, a_s and τ_s can be estimated from the linear regression of $log(1 - \frac{T_1}{ECS})$ versus t for the quadrupled CO_2 experiments. $a_f = 1 - a_s$, and τ_f can be expressed as

$$\tau_f = t / \left[log(a_f) - log(1 - T_1/ECS - a_s e^{-t/\tau_s}) \right], \tag{3}$$

which can be estimated by averaging over the first ten years of the step-forcing experiment. The heat capacities and γ can be computed from the other parameters.

The parameters are then adjusted iteratively, using the EBM. For each iteration, a multi-linear regression of $N = c\frac{dT_1}{dt}$ is performed, using the ocean heat uptake $H = \gamma(T_1 - T_2)$

from the previous iteration:

$$N = F_i - \lambda_i T_1 - (\epsilon_i - 1) H_{i-1}, \tag{4}$$

where the time-series of N and T are taken from the $4XCO_2$ experiments.

Integrations of the EBM were performed using backwards Euler time-stepping, with a time-step of one year. The results are not sensitive to the choice of time-step or the choice of numerical integration scheme.

Text S2. Fitting the 20th Century

The 20th century surface temperature is taken from the HADCRUT4 dataset, available at https://crudata.uea.ac.uk/cru/data/temperature/, and the 20th century net radiative forcing and CO_2 concentrations were downloaded from http://www.pik-potsdam.de/∼mmalte/rcps/. The radiative forcing due to CO_2 was again calculated as $\Delta F_{CO2}(t) = F_{CO2} log(C(t)/C_0)$.

To find the optimal value of λ the calculations were repeated for all values of λ between 0.01 and $3\text{Wm}^{-2}\text{K}^{-1}$, in increments of $0.01\text{Wm}^{-2}\text{K}^{-1}$, and then the optimal value was taken as the value of λ which produced the time-series of T_1 with the smallest root-mean-squared error compared to the observed temperature record.

References

Geoffroy, O., D. Saint-Martin, G. Bellon, A. Voldoire, D. J. L. Olivie, and S. Tyteca (2013a), Transient climate response in a two-layer energy-balance model. part i: Analytical solution and parameter calibration using cmip5 aogcm experiments, *Journal of Climate*, *26*(6), 1841–1859.

Gregory, J. M. , W. J. Ingram, M. A. Palmer, G. S. Jones, P. A. Stott, R. B. Thorpe, J. A. Lowe, T. C. Johns, and K. D. Williams. (2004), A new method for diagnosing radiative forcing and climate sensitivity., *Geophysical Research Letters*, *31*(9), L03205

Table S1. Definitions of terms in analytic solution to the EBM.

Fast terms:	Slow terms:
$a_f = \frac{\phi_s \tau_f}{c(\phi_s - \phi_f)}\lambda$ $\phi_f = \frac{c}{2\gamma\epsilon}(b^* - \sqrt{\delta})$ $\tau_f = \frac{cc_0}{2\lambda\gamma}(b - \sqrt{\delta})$	$a_s = \frac{\phi_f \tau_s}{c(\phi_s - \phi_f)}\lambda$ $\phi_s = \frac{c}{2\gamma\epsilon}(b^* + \sqrt{\delta})$ $\tau_s = \frac{cc_0}{2\lambda\gamma}(b + \sqrt{\delta})$
Other terms:	
$b = \frac{\lambda + \gamma\epsilon}{c} + \frac{\gamma}{c_0}$ $b^* = \frac{\lambda + \gamma\epsilon}{c} - \frac{\gamma}{c_0}$ $\delta = b^2 - 4\frac{\lambda\gamma}{cc_0}$	

Table S2. The models used in this study and their corresponding values of F (units = $\mathrm{Wm^{-2}}$), λ ($\mathrm{Wm^{-2}K^{-1}}$), c ($\mathrm{Wm^{-2}K^{-1}s^{-1}}$), c_0 ($\mathrm{Wm^{-2}K^{-1}s^{-1}}$) γ ($\mathrm{Wm^{-2}K^{-1}}$) and ϵ (dimensionless), as well as each model's TCR (K) and ECS (K).

Model	F	λ	c	c_0	γ	ϵ	TCR	ECS
BCC-CSM1-1	3.7	1.28	8.4	56	0.59	1.27	1.7	2.9
BNU-ESM	3.65	0.92	7.3	89	0.54	0.92	2.2	4.0
CanESM2	4.1	1.06	8.0	77	0.54	1.28	2.4	3.9
CNRM-CM5	3.6	1.12	8.3	95	0.51	0.92	2.1	3.2
CSIRO-Mk3-6-0	3.5	0.68	8.5	76	0.71	1.82	1.8	5.1
FGOALS-s2	4.0	0.87	7.5	138	0.72	1.21	2.4	4.5
GFDL-CM3	3.4	0.81	8.8	74	0.82	1.36	2.0	4.1
GFDL-ESM2G	3.55	1.49	6.2	98	0.83	1.15	1.1	2.4
GFDL-ESM2M	3.55	1.38	8.8	112	0.84	1.21	1.3	2.5
GISS-ESM-LR	4.5	2.03	6.1	134	1.06	1.44	1.5	2.2
HADGEM2-ES	3.4	0.61	7.5	98	0.59	1.54	2.5	4.5
INMCM4	3.0	1.56	8.5	271	0.54	0.83	1.3	2.1
IPSL-CM5A-LR	3.4	0.79	8.1	100	0.57	1.14	2.0	4.1
MIROC5	4.5	1.58	8.7	158	0.73	1.19	1.5	2.7
MPI-ESM-LR	4.7	1.21	8.5	78	0.62	1.42	2.0	3.7
MRI-CGCM3	3.5	1.31	9.3	68	0.59	1.25	1.6	2.6
NCAR-CCSM4	4.2	1.4	7.6	72	0.81	1.36	1.8	2.9
NorESM1-M	3.7	1.18	9.7	121	0.76	1.57	1.4	2.8

Table S3. r^2 for correlations across models between the six parameters of the EBM.

r^2	F	λ	γ	ϵ	c	c_0
F	1					
λ	0.17	1				
γ	0.1	0.37	1			
ϵ	0.07	0.07	0.06	1		
c	0.05	0.06	0.09	0.01	1	
c_0	0.04	0.15	0.04	0.19	0.04	1

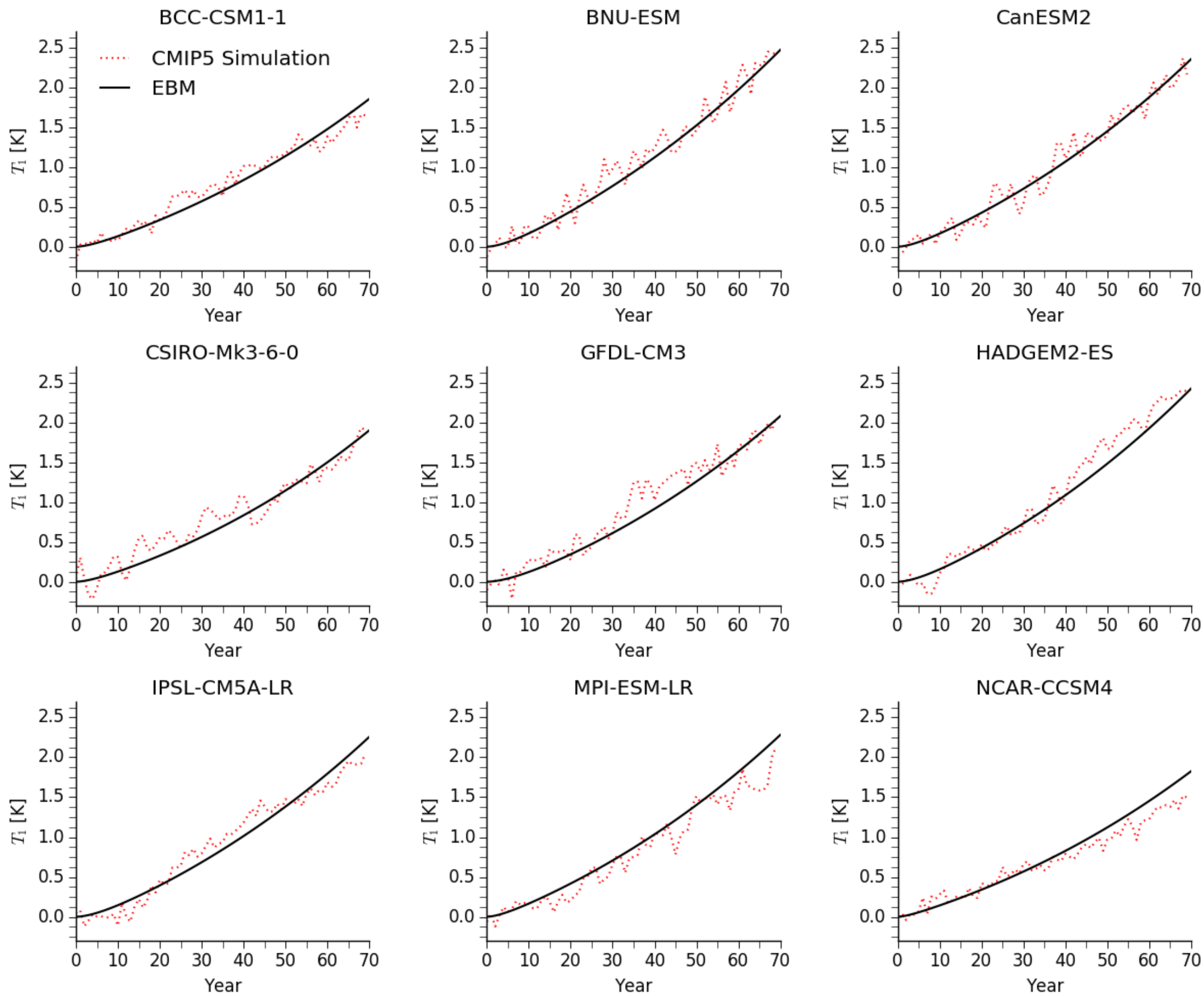

Figure S1. Time series of anomalous global-mean surface temperature in the 1% CO_2 simulations with nine of the models analyzed in this study (dotted red lines) and corresponding simulations with the EBM (solid black lines).